The Closing Bracket . . .

Two themes stand out in this issue: that of proofs without words, or PWWs as they are generally referred to, and that of algorithms for school-level arithmetical operations and manipulations and their hidden basis. The former serves to bring out the 'fun' and visually appealing side of the subject by designing compact and pretty pictures that drive home some point. The latter points to the deep interconnectedness of the subject by showing that familiar operations and algorithms of elementary arithmetic typically have roots that lie deep within the structure of the subject.

Very few students get to see PWWs, and perhaps fewer still are challenged to construct or devise pictures of their own which serve to illustrate some notion. Yet it appears to me that it may be a wonderful way to learn something of value by getting them to ponder the matter at hand, slowly and with a sense of leisure, searching for ways of depicting patterns and structures. It is necessary to bring in the element of slowness into math education (and into all of education, more generally), and devising pictures is one possible way to approach this challenge.

Similarly, very few students get to see and experience the interconnectedness of the subject. High quality exposition may be needed to bring out this factor, but one possible approach is through non-routine problem solving. The problems have to be chosen with care, but they can serve as excellent platforms for bringing in multiple strands together and seeing interconnections.

Here is a quote from Paul Halmos, the great Hungarian-born US mathematician who was an equally great teacher: *It is the duty of all teachers, and of teachers of mathematics in particular, to expose their students to problems much more than to facts*. PWWs would surely qualify for the description "much more than facts"; so also the matter of unearthing interconnectedness. Of course that phrase includes a lot more than that: it also includes the experience of conjecturing, of experimentation, of testing your conjectures, of devising counterexamples, of possibly finding that your conjectures are wrong, of proving them. It is only when one experiences the subject in this complete sense that one begins to see the wholeness of the subject. All students of mathematics have the right to experience the subject in that way, and we surely have to make it so for them. (Here's another quote from Halmos: "Mathematics is not a deductive science — that's a cliché. When you try to prove a theorem, you don't just list the hypotheses, and then start to reason. What you do is trial and error, experimentation, guesswork." Paul Halmos certainly had a way with words.)

Halmos was a great proponent of the so-called "Moore method" for teaching mathematics. But let's leave that theme for a future issue.

— Shailesh Shirali